Reg. No.						

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal
Odd Semester, 2013 - 2014

MA939 Functional Analysis

Problem Sheet - 2

Date : 26.07.2013 Last Date of Submission : 29.07.2013

Answer **ALL** questions.

1. Give an example of an unbounded metric on a set which is translation invariant but not absolutely homogeneous.
2. Give an example of an unbounded metric on a set which is absolutely homogeneous but not translation invariant.
3. If X is an infinite dimensional normed space with respect to the norm (constructed earlier to prove that every linear space can be normed), then prove the X is not Banach with respect to the norm.
4. A non-zero normed space X is Banach iff its unit sphere centered at the origin is complete.
5. Whether Holder's inequality for both n-tuples and sequences, is true for $p = 1$ and $p = \infty$?
6. Prove that c and c_0 are closed subspaces of ℓ_∞.
7. Prove the Minkowski's inequality for sequences.
8. Prove that if $x \in \ell_1$, $\|x\|_1 \geq \|x\|_2 \geq \cdots \geq \|x\|_\infty$.
9. Prove the Minkowski's inequality for integrals.
10. Give an example of two subsets A and B of the set of real numbers such that $A+B$ is not closed.
11. If E is of finite measure and $1 \leq p < \infty$, then prove that $L_\infty(E)$ is a subspace of $L_p(E)$.
12. Check and find an example of a set which is neither nowhere dense nor everywhere dense.
13. Every complete metric space is of second category. What about the converse?
14. Say true or false: Every subspace of a normed space is everywhere dense or nowhere dense.