Reg. No.						

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal
Odd Semester, 2013 - 2014
MA939 Functional Analysis
Problem Sheet - 3

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Answer **ALL** questions.

1. Prove that the space ℓ_p for any $1 \leq p < \infty$ is not closed in ℓ_∞.
2. What is the smallest closed subspace containing c_{00} in ℓ_∞?
3. Explain the relations between bounded sets and totally bounded sets of a metric space X. What is the case if $X = \mathbb{K}^n$?
4. Prove that $C^k[a,b]$ is a proper dense subspace of $L_p[a,b]$ with $\|.\|_p$ for $1 \leq p < \infty$.
5. Prove that $C^k[a,b]$ is not Banach with respect to any norm $\|.\|_p$ for $1 \leq p \leq \infty$ but it is Banach with respect to $\sum_{j=0}^{k} \|x^{(j)}\|_\infty$.
6. The space $\mathcal{P}[a,b]$ of all polynomials defined on $[a,b]$ is not Banach with respect to any norm.
7. If $X = \mathbb{N}$ with the discrete metric, prove that $C(X) = \ell_\infty$.
8. For $1 \leq p < \infty$, using the convexity of the function $f(t) = t^p$ on $[0,\infty)$, show that ℓ_p is a vector space.
9. $C^\infty[a,b]$ is not Banach in the induced norm of $C^k[a,b]$ (the complete norm as defined above).
10. It is given that $C_c(X)$ is not a closed subspace of $C(X)$ so that $C_c(X)$ is not Banach with respect to sup norm. Find a sequence (x_n) in $C_c(X)$ which converges to x in $C(X)$ but $x \notin C_c(X)$.