Department of Mathematical and Computational Sciences
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Numerical Analysis - MA 704
Problem Sheet 7

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1. The following values of function are given.

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	1.543	1.668	1.811	1.971	2.151	2.352	2.577	2.828	3.107

   (a) Find $\int_1^{1.8} f(x)dx$ using the trapezoidal rule with (i) $h = 0.1$, (ii) $h = 0.2$, (iii) $h = 0.4$
   (b) Estimate the error in the computed value in each case.
   (c) Extrapolate the individual answers to get estimates of improved accuracy using Romberg integration.

2. Compute $\int_0^1 \frac{dx}{\sqrt{x^4+1}}$ by Gauss quadrature formula, given the following data.

Values of x	Weighting factor
0.0	0.88888889
$\pm$0.77459667	0.55555555

3. (a) Using Adam-Bashforth predictor corrector method, obtain the solution of $\frac{dy}{dx} = x - y^2$ at $x = 0.8$ correct to 3 decimal places given the values:

x	0	0.2	0.4	0.6
y	0	0.0200	0.0795	0.1762

   (b) Using Runge-Kutta method of order 4, compute $y(0.1)$ given $\frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right) - 4y = 0$ subject to $y = 0.2$, $\frac{dy}{dx} = 0.5$ at $x = 0$.

4. (a) Derive the Standard five-point formula for the Laplace equation.
   (b) Solve

$$\begin{aligned} u_{xx} + u_{yy} &= 0 \qquad \text{in } 0 < x < 1, 0 < y < 1 \\ u(x,1) &= 0 \\ u(0,y) &= 0 \\ u(1,y) &= 9(y - y^2), u(x,0) = 9(x - x^2) \end{aligned}$$

   with $h = k = \frac{1}{3}$.

5. Derive the Crank-Nicholson scheme and hence solve $u_t = u_{xx}$ subject to

$$u(x,0) = \sin \pi x, 0 \leq x \leq 1, u(0,t) = u(1,t) = 0, t \geq 0$$

   with $h = \frac{1}{4}$ for two time levels. Choosing $k = \frac{1}{32}$.

6. (a) Solve the wave equation $u_{tt} = u_{xx}, 0 \leq x \leq 1$, for $0 \leq t \leq 0.4$ with the boundary conditions $u(0,t) = u(1,t) = 0$, assuming initial deflection $f(x) = 1 - \cos 2\pi x$ and initial velocity is zero and choosing $h = k = 0.2$.
   (b) Solve the following boundary value problem using Galerkin method$u'' + u = -x, 0 \leq x \leq 1$ with $u(0) = u(1) = 0$ with the approximate solution $w(x) = x(1-x)(a_1 + a_2 x)$.