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Concrete Mathematics - MA 201
Problem Sheet - 2

1. Verify that the following are equivalent :
   (a) The nth line (for $n \geq 0$) increases the number of regions by k.
   (b) The nth line splits k of the old regions.
   (c) The nth line hits the previous lines in $k-1$ different places.

2. Let L_n denote the maximum number of regions defined by n lines in a plane. Prove

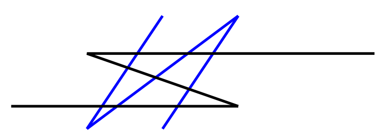
$$L_n = 1 + \frac{n(n+1)}{2}, \qquad \text{for} \quad n \geq 0$$

   by induction.

3. For large value of n, can we say that there are four times as many regions with bent lines as with straight lines?

4. Some of the regions defined by n lines in the plane are infinite, while others are bounded. What is the maximum possible number of bounded regions?

5. What is the maximum number of regions definable by n zig-zag lines, each of which consists of two parallel infinite half-lines joined by a straight segment?



6. How many pieces of cheese can you obtain from a single thick piece by making five straight slices? (The cheese must stay in its original position while you do all the cutting, and each slice must correspond to a plane in $3D$.) Find a recurrence relation for P_n, the maximum number of three-dimensional regions that can be defined by n different planes.

7. Show that the following set of n bent lines defines Z_n regions, where Z_n is defined by

$$Z_n = 2n^2 - n + 1, \quad \text{for} \quad n \geq 0.$$

   The jth bent line, for $1 \leq j \leq n$, has its zig at $(n^{2j}, 0)$ and goes up through the points $(n^{2j} - n^j, 1)$ and $(n^{2j} - n^j - n^{-n}, 1)$.

8. Is it possible to obtain Z_n regions with n bent lines when the angle at each zig is $30°$?

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